KCET EXAMINATION – 2022
SUBJECT : MATHEMATICS (VERSION – D3)

DATE :- 16-06-2022 **TIME : 02.30 PM TO 03.50 PM**

1. The octant in which the point (2, -4, -7)
   1) Eight 2) Third
   3) Fourth 4) Fifth

Ans. 1

Sol. Conceptual

2. If $f(x)=\begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3 \end{cases}$ the quadratic equation whose roots are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ is
   1) $x^2-14x+49=0$ 2) $x^2-10x+21=0$
   3) $x^2-6x+9=0$ 4) $x^2-7x+8=0$

Ans. 2

Sol. $\alpha=\lim_{x\to 2^-} f(x)=\lim_{x\to 2} x^2-1=3$

$\beta=\lim_{x\to 2^+} f(x)=\lim_{x\to 2} 2x+3=7$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-10x+21=0$

3. If $3x+i(4x-y)=6-i$ where x and y are real numbers, then the values of x and y respectively,
   1) 3, 9 2) 2, 4 3) 2, 9 4) 3, 4

Ans. 3

Sol. $3x=6$

$x=2$ $4x-y=-1$

$8-y=-1$

$9=y$

4. If all permutations of the letters of the word MASK are arranged in the order as in dictionary without meaning, which one of the following is 19th word
   1) KAMS 2) SAMK 3) AKMS 4) AMSK

Ans. NO OPTION

Sol. Original answer SAKM

A K M S

$A\to 3!$

$K\to 3!$

$M\to 3!$

$SAKM\to \frac{1}{19}$

5. If $a_1,a_2,a_3,.....a_{10}$ is a geometric progression and $\frac{a_3}{a_1}=25$, then $\frac{a_9}{a_5}$ equals
   1) $3(5^2)$ 2) 5^4 3) 5^3 4) $2(5^2)$

Ans. 2

Sol. $\frac{a_3}{a_1}=25$

$\frac{ar^2}{a}=25$

$r^2=5^2$

$\frac{a_4}{a_5}=\frac{ar^8}{ar^4}=r^4=5^4$

6. If the straight line 2x-3y+17=0 is perpendicular to the line passing through the point $(7,17)$ and $(15,\beta)$, then β equals
   1) -5 2) 5 3) 29 4) -29

Ans. 2

Sol. $m_1\times m_2=-1$

$\frac{2}{3}\times\frac{\beta-17}{15-7}=-1$

$\beta-17=-12$

$\beta=5$

7. Let the relation R is defined in N by aRb, if 3a+2b=27 then R is

1) $\{(1,12)(3,9)(5,6)(7,3)\}$

2) $\left\{\left(0,\dfrac{27}{2}\right)(1,12)(3,9)(5,6)(7,3)\right\}$

3) $\{(1,12)(3,9)(5,6)(7,3)(9,0)\}$

4) $\{(2,1)(9,3)(6,5)(3,7)\}$

Ans. 1

Sol. $2b = 27 - 3a$

$b = \dfrac{27-3a}{2}$

$R = \{(1,2),(3,9),(5,6),(7,3)\}$

8. $\lim\limits_{y\to 0}\dfrac{\sqrt{3+y^3}-\sqrt{3}}{y^3} =$

1) $\dfrac{1}{2\sqrt{3}}$ 2) $\dfrac{1}{3\sqrt{2}}$ 3) $2\sqrt{3}$ 4) $3\sqrt{2}$

Ans. 1

Sol. $\lim\limits_{y\to 0}\dfrac{3+y^3-3}{y\left(\sqrt{3+y^3}+\sqrt{3}\right)} = \dfrac{1}{2\sqrt{3}}$

9. If the standard deviation of the numbers $-1,0,1,k$ is $\sqrt{5}$ where k>0, then k is equal to

1) $4\sqrt{\dfrac{5}{3}}$ 2) $\sqrt{6}$ 3) $2\sqrt{\dfrac{10}{3}}$ 4) $2\sqrt{6}$

Ans. 4

Sol. $\sigma^2 = 5$, $\bar{x} = \dfrac{k}{4}$

$\dfrac{1}{4}\left(1+0+1+k^2\right) - \dfrac{k^2}{16} = 5$

$\dfrac{k^2+2}{4} - \dfrac{k^2}{16} = 5$

$\dfrac{4k^2+8-k^2}{16} = 5 \Rightarrow 3k^2+8 = 80$

$3k^2 = 72$

$k^2 = 24$

$k = \pm\sqrt{24} = 2\sqrt{6}$

10. If the set X contains 7 elements set y contains 8 elements, then the number of bijections from X to Y is

1) 0 2) $8P_7$ 3) 7! 4) 8!

Ans. 1

Sol. $n(A) \neq n(B)$

Number of bijections is zero

11. If f:R→R be defined by $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \le 3 \\ 3x : x \le 1 \end{cases}$

then $f(-1)+f(2)+f(4)$ is

1) 5 2) 10 3) 9 4) 14

Ans. 3

Sol. $f(-1) = 3(-1) = -3$

$f(2) = 2^2 = 4$

$f(4) = 2(4) = 8$

$f(-1)+f(2)+f(4) = -3+4+8 = 9$

12. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then $(aI+bA)^n$ is (where 1 is the identify matrix of order 2)

1) $a^2I + a^{n-1}b.A$ 2) $a^nI + n.a^{n-1}b.A$

3) $a^nI + n\,a^nbA$ 4) $a^nI + b^nA$

Ans. 2

Sol. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$[aI+bA]^1 = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$[aI+IA]^2 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$

$[aI+bA]^3 = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix}$

$\therefore [aI+bA]^n = \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix} = a^nI + n.a^{n-1}bA$

13. If A is a 3×3 matrix such that $|5.\text{adj}A| = 5$ then $|A|$ is equal to

1) ± 1 2) $\pm 1/25$ 3) $\pm 1/5$ 4) ± 5

Ans. 3

Sol. $A_{3\times 3}$ matrix $|5.\text{Adj}A| = 5$

$\Rightarrow 5^3|A|^2 = 5 \Rightarrow |A^2| = \dfrac{1}{5^2}$

$|A| = \pm\dfrac{1}{5}$

14. If there are two value of 'a' which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$.

Then the sum of these numbers is

1) -4 2) 9 3) 4 4) 5

Ans. 1

Sol. $\Delta = 1\left(2a^2+4\right) + 2(4a-0) + 5(8) = 86$

$2a^2 + 8a + 44 - 86 = 0$

$2a^2 + 8a - 42 = 0$

$a^2 + 4a - 21 = 0$

Sum of numbers=-4 $\left(\therefore -\frac{b}{a} = \alpha + \beta\right)$

15. If the vertices of a triangle are $(-2,6)(3,-6)$ and $(1,5)$, then the area of the triangle is

1) 40 sq.units 2) 15.5 sq.units
3) 30 sq.units 4) 35 sq.units

Ans. 2

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} -2-3 & -2-1 \\ 6+6 & 6-5 \end{vmatrix} = \frac{1}{5}\begin{vmatrix} -5 & -3 \\ 12 & 1 \end{vmatrix}$

$= \frac{1}{2}|-5+36| = \frac{31}{2} = 15.5$

16. Domain of $\cos^{-1}[x]$ is, where [.] denotes a greatest integer function

1) $(-1,2]$ 2) $(-1,2)$ 3) $[-1,2]$ 4) $[-1,2)$

Ans. 4

Sol. $\cos^{-1}[x]$

$-1 \le [x] \le 1 \Rightarrow$ [x]= {-1, 0, 1}

$x \in [-1,2)$

17. If A is a matrix of order 3×3, then $(A^2)^{-1}$ is equal to

1) $(-A^2)^2$ 2) $(A^{-1})^2$ 3) A^2 4) $(-A)^{-2}$

Ans. 2

Sol. $(A^2)^{-1} = (A^{-1})^2$

18. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then the inverse of the matrix A^3 is

1) A 2) -I 3) I 4) -A

Ans. 1

Sol. $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = A$

$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore A^3 = A$

19. If A is a skew symmetric matrix, then A^{2021} is

1) Row matrix
2) Column matrix
3) Symmetric matrix
4) Skew symmetric matrix

Ans. 4

Sol. $A^T = -A$ or A^n is slow symmetric if n is odd

$P = A^{2021}$

$P^T = \left[A^{2021}\right]^T = \left[A^T\right]^{2021} = (-A)^{2021} = -P$

20. If $f(1) = 1, f'(1) = 3$ then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

1) 10 2) 33 3) 35 4) 12

Ans. 2

Sol. $f(1) = 1,\ f'(1) = 3$

$\frac{d}{dx}\left[f(f(f(x))) + (f(x))^2\right]$

$\left[f'(f(f(x))).f'f(x).f'(x) + 2f(x).f'(x)\right]$

$= f'(f(f(1)))f'(f(1)).f'(1) + 2f(1).f'(1)$

$f'(f(1))f'(1).3 + 2.(1)3$

$= f'(1).3.3 + 6$

= 27+6 = 33

21. If $y = x^{\sin x} + (\sin x)^x$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

1) $\frac{4}{\pi}$ 2) $\pi \log\frac{\pi}{2}$ 3) 1 4) $\frac{\pi^2}{2}$

Ans. 3

Sol. $y = x^{\sin x} + (\sin x)^x$

$\frac{dy}{dx} = \left[x^{\sin x}\right]\left[\frac{\sin x}{x} + \cos x.\log x\right] +$

$(\sin x)^x\left[x\cos x + \log\sin x\right]$

$x = \frac{\pi}{2}$

$= \frac{\pi}{2}\left[\frac{2}{\pi}\right] + 1[0+0] = 1$

22. If $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$ then $|A_1| + |A_2| + ... + |A_{2021}| =$

1) -2021 2) $-(2021)^2$
3) $(2021)^2$ 4) 4042

Ans. 2

Sol. $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$

$|A_n| = (1-n)^2 - n^2$

$= 1 + n^2 - 2n - n^2$

23. If $y=\left(1+x^2\right)\tan^{-1}x-x$ then $\frac{dy}{dx}$ is

1) $2x\tan^{-1}x$ 2) $\frac{\tan^{-1}x}{x}$

3) $x^2\tan^{-1}x$ 4) $x\tan^{-1}x$

Ans. 1

Sol. $y=\left(1+x^2\right)\tan^{-1}x-x$

$$\frac{dy}{dx}=\frac{\left(1+x^2\right)}{1+x^2}+\tan^{-1}x.(2x)-1$$

$$=2x\tan^{-1}x$$

24. If $x=e^{\theta}\sin\theta, y=e^{\theta}\cos\theta$ where θ is a parameter, then $\frac{dy}{dx}$ at $(1,1)$ is equal to

1) 0 2) $\frac{1}{2}$ 3) $-\frac{1}{2}$ 4) $-\frac{1}{4}$

Ans. 1

Sol. $x=e^{\theta}\sin\theta=1$

$y=e^{\theta}\cos\theta=1$, $\frac{x}{y}=\tan\theta=1\Rightarrow\theta=\frac{\pi}{4}$

$$\frac{dy}{dx}=\left|\frac{dy/d\theta}{dx/d\theta}\right|=\frac{-e^{\theta}\sin\theta+\cos\theta.e^{\theta}}{e^{\theta}\cos\theta+\sin\theta e^{\theta}}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$=\tan\left(\frac{\pi}{4}-\theta\right)=0$$

25. If $y=e^{\sqrt{x\sqrt{x\sqrt{x}}}\,......}, x>1$ then $\frac{d^2y}{dx^2}$ at $x=\log_e^3$ is

1) 3 2) 5 3) 0 4) 1

Ans. 1

Sol. $y=e^{\sqrt{x\sqrt{x\sqrt{x}}}\,....}=e^{x^{\frac{1}{2}}x^{\frac{1}{4}}.x^{\frac{1}{8}}......}=e^{x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}}$

$$e^{x^{\frac{1}{2}\left[1+\frac{1}{2}+\frac{1}{4}+....\right]}}=e^{x^{\frac{1}{2}\cdot 2}}=e^{x^1}=e^x$$

$$\frac{dy}{dx}=e^x$$

$$\frac{d^2y}{dx^2}=e^x x\log_e^3=e^{\log_e^3}=$$

26. If $[x]$ is the greatest integer function not greater than x then $\int_0^8[x]\,dx$ is equal to

1) 28 2) 30 3) 29 4) 20

Ans. 1

Sol. $\int_0^8[x]dx=1+2+3+....+7$

$$=\frac{7(7+1)}{2}=28$$

27. $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\cos^3\theta\,d\theta$ is equal to

1) $\frac{8}{23}$ 2) $\frac{7}{23}$ 3) $\frac{8}{21}$ 4) $\frac{7}{21}$

Ans. 3

Sol. Put $\sin\theta=t$

$$\int_0^1 t^{1/2}(1-t^2)dt=\frac{8}{21}$$

28. If $e^y+xy=e$ the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at $x=0$ is equal to

1) $\left(\frac{1}{e},\frac{1}{e^2}\right)$ 2) $\left(\frac{-1}{e},\frac{-1}{e^2}\right)$

3) $\left(\frac{1}{e},\frac{-1}{e^2}\right)$ 4) $\left(\frac{-1}{e},\frac{1}{e^2}\right)$

Ans. 4

Sol. $x=0\Rightarrow y=1$

$$\frac{dy}{dx}=\frac{-y}{e^y+x}$$

$$\left(\frac{dy}{dx}\right)_{(0,1)}=-\frac{1}{e}$$

$$\left(\frac{d^2y}{dx^2}\right)_{(0,1)}=\frac{1}{e^2}$$

29. The function $f(x)=\log(1+x)-\frac{2x}{2+x}$ is increasing on

1) $(-\infty,\infty)$ 2) $(\infty,-1)$ 3) $(-1,\infty)$ 4) $(-\infty,0)$

Ans. 3

Sol. $f'(x)=\frac{x^2}{(x+1)(2+x)^2}>0$

$x+1>0\Rightarrow x>-1$

30. The co-ordinates of the point on the $\sqrt{x}+\sqrt{y}=6$ at which the tangent is equally inclined to the axes is

1) $(4,4)$ 2) $(1,1)$ 3) $(9,9)$ 4) $(6,6)$

Ans. 3

Sol. $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}=-1$

$y=x$

$\sqrt{x}+\sqrt{x}=6$

$x=9$, $y=9$

31. The function $f(x)=4\sin^3 x-6\sin^2 x+12\sin x+100$ is strictly

1) decreasing in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

2) decreasing in $\left[0,\frac{\pi}{2}\right]$

3) increasing in $\left(\pi,\frac{3\pi}{2}\right)$

4) decreasing in $\left(\frac{\pi}{2},\pi\right)$

Ans. 4

Sol. $f'(x)=(12\sin^2 x-12\sin x+12)\cos x$

$f'(x)=12(\sin^2 x-\sin x+1)\cos x$

$\sin^2 x-\sin x+1>0$

$x\in\left(\frac{\pi}{2},\pi\right)\quad \cos x<0$

32. Area of the region bounded by the curve $y=\tan x$, the $x-$axis and the line $x=\frac{\pi}{3}$ is

1) $\log\frac{1}{2}$ 2) $\log 2$ 3) 0 4) $-\log 2$

Ans. 2

Sol. $A=\int_0^{\pi/3}\tan x\,dx=\log|\sec x|_0^{\pi/3}=\log 2$

33. Evaluate $\int_2^3 x^2dx$ as the limit of a sum

1) $\frac{72}{6}$ 2) $\frac{53}{9}$ 3) $\frac{25}{7}$ 4) $\frac{19}{3}$

Ans. 4

Sol. $I=\left[\frac{x^3}{3}\right]_2^3=\frac{1}{3}(27-8)=\frac{19}{3}$

34. $\int_0^{\frac{\pi}{2}}\frac{\cos x\sin x}{1+\sin x}dx$ is equal to

1) $\log 2-1$ 2) $\log 2$

3) $-\log 2$ 4) $1-\log 2$

Ans. 4

Sol. Sinx=t

$\int_0^{\frac{\pi}{2}}\frac{\cos x\sin x}{1+\sin x}dx\Rightarrow\int_0^1\frac{t}{1+t}dt=1-\log 2$

35. $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$ is equal to

1) $2(\sin x-x\cos\alpha)+c$

2) $2(\sin x+x\cos\alpha)+c$

3) $2(\sin x-2x\cos\alpha)+c$

4) $2(\sin x+2x\cos\alpha)+c$

Ans. 2

Sol. $2\int\frac{\cos^2 x-\cos^2\alpha}{\cos x-\cos\alpha}dx=2\int(\cos x+\cos\alpha)dx$

$=2[\sin x+x\cos\alpha]$

36. $\int_0^1\frac{xe^x}{(2+x)^3}dx$ is equal to

1) $\frac{1}{27}.e-\frac{1}{8}$ 2) $\frac{1}{27}.e+\frac{1}{8}$

3) $\frac{1}{9}.e+\frac{1}{4}$ 4) $\frac{1}{9}.e-\frac{1}{4}$

Ans. 4

Sol. $\int_0^1 e^x\left[\frac{1}{(x+2)^2}-\frac{2}{(x+2)^3}\right]dx$

$=\left[\frac{e^x}{(x+2)^2}\right]_0^1=\frac{e}{9}-\frac{1}{4}$

37. If $\int\frac{dx}{(x+2)(x^2+1)}=$

$a\log|1+x^2|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+c$, then

1) $a=\frac{-1}{10},b=\frac{2}{5}$ 2) $a=\frac{1}{10},b=\frac{2}{5}$

3) $a=\frac{-1}{10},b=\frac{-2}{5}$ 4) $a=\frac{1}{10},b=\frac{-2}{5}$

Ans. 1

Sol. $\frac{1}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}$

$A=\frac{1}{5},B=\frac{-1}{5},C=\frac{2}{5}$

38. If $|\vec{a}|=2$ and $|\vec{b}|=3$ and the angle between $\vec{a}$ and $\vec{b}$ is 120^0, then the length of the vector $\left|\frac{1}{2}\vec{a}-\frac{1}{3}\vec{b}\right|^2$ is

1) 2 2) 3 3) $\frac{1}{6}$ 4) 1

ORIGINAL QUESTION

38. If $|\vec{a}|=2$ and $|\vec{b}|=3$ and the angle between $\vec{a}$ and $\vec{b}$ is 120^0, then the length of the vector $\left|\frac{\vec{a}}{2}-\frac{\vec{b}}{3}\right|^2$ is

Ans. **2**

Sol. $\left|\frac{\vec{a}}{2}-\frac{\vec{b}}{3}\right|^2=\frac{|\vec{a}|^2}{4}+\frac{|\vec{b}|^2}{9}-2\frac{\vec{a}}{2}.\frac{\vec{b}}{3}=3$

39. If $|\vec{a}\times\vec{b}|+|\vec{a}.\vec{b}|^2=36$ and $|\vec{a}|=3$ then $|\vec{b}|$ is equal to

1) 9 2) 36 3) 4 4) 2

ORIGINAL QUESTION

39. If $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2=36$ and $|\vec{a}|=3$ then $|\vec{b}|$ is equal to

Ans. **4**

Sol. $=|a\times b|^2+|a-b|^2=36$

$|a|^2|b|^2=36\Rightarrow b^2=4,|b|=2$

40. If $\vec{\alpha}=\hat{i}-3\hat{j}$, $\vec{\beta}=\hat{i}+2\hat{j}-\hat{k}$ then express $\vec{\beta}$ in the from $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$ then $\vec{\beta}_1$ is given by

1) $\frac{5}{8}(\hat{i}-3\hat{j})$ 2) $\frac{5}{8}(\hat{i}+3\hat{j})$

3) $\hat{i}-3\hat{j}$ 4) $\hat{i}+3\hat{j}$

Ans. **NO OPTION**

Sol. **Correct answer is** $-\frac{1}{2}(\hat{i}-3\hat{j})$

$\vec{\beta}=(\vec{\beta}_1+\vec{\beta}_2)$

$(\vec{\beta}=\lambda\vec{\alpha}+\vec{\beta}_2).\vec{\alpha}$

$\vec{\alpha}.\vec{\beta}=\lambda|\vec{\alpha}|^2+0$

41. Then sum of the degree and order of the differential equation $(1+y_1^2)^{2/3}=y_2$ is

1) 4 2) 6 3) 5 4) 7

Ans. **3**

Sol. $(1+y_1^2)=(y_2)^3$

$2+3=5$

42. If $\frac{dy}{dx}+\frac{y}{x}=x^2$ then $2y(2)-y(1)=$

1) $\frac{11}{4}$ 2) $\frac{15}{4}$ 3) $\frac{9}{4}$ 4) $\frac{13}{4}$

Ans. **2**

Sol. $y.x=\frac{x^4}{4}+C$

$2y(2)-y(1)=\frac{15}{4}$

43. The solution of the differential equation $\frac{dy}{dx}=(x+y)^2$ is

1) $\tan^{-1}(x+y)=x+c$

2) $\tan^{-1}(x+y)=0$

3) $\cot^{-1}(x+y)=c$

4) $\cot^{-1}(x+y)=x+c$

Ans. **1**

Sol. $x+y=z\Rightarrow\frac{dz}{dx}=1+z^2$

$\int\frac{1}{1+z^2}dz=\int 1dx$

$\text{Tan}^{-1}(x+y)=x+c$

44. If $y(x)$ be the solution of differential equation $x\log x\frac{dy}{dx}+y=2x\log x$, $y(e)$ is equal to

1) e 2) 0 3) 2 4) 2e

Ans. **4**

Sol. I.F= logx,

$y\log x=2x(\log x-1)+c$

If x =e then y=c then y(e)=2e

45. A dietician has to develop a special diet using two foods X and Y. Each packet (containing 30g) of food. X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and most 300 units of cholesterol. The corner points of the feasible region are

1) (2, 72), (40, 15), (15, 20)
2) (2, 72), (15, 20), (0, 23)
3) (0, 23), (40, 15), (2, 72)
4) (2, 72), (40, 15), (115, 0)

Ans. **1**

Sol.

46. The distance of the point position vector is $(2\hat{i}+\hat{j}-\hat{k})$ from the plane $\vec{r}.(\hat{i}-2\hat{j}+4\hat{k})=4$ is

1) $\frac{8}{\sqrt{21}}$ 2) $8\sqrt{21}$ 3) $-\frac{8}{\sqrt{21}}$ 4) $\frac{-8}{21}$

Ans. 1

Sol. $\text{Distance} = \frac{|2-2-4-4|}{\sqrt{1+4+16}} = \frac{(8)}{\sqrt{21}}$

47. The co-ordinate of foot of the perpendicular drawn from the origin to the plane $2x-3y+4z=29$ are

1) (2, 3, 4) 2) (2, -3, -4)
3) (2, -3, 4) 4) (-2, -3, 4)

Ans. 3

Sol. verification (2, -3, 4)

48. The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

1) $\theta=\cos^{-1}\left[\frac{27}{5}\right]$ 2) $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$

3) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$ 4) $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$

Ans. NO OPTION

Sol. Original answer

$$\cos\theta=\frac{3\times1+5\times4+4\times2}{\sqrt{3^2+5^2+4^2}\times\sqrt{1^2+4^2+2^2}}=\frac{31}{5\sqrt{42}}$$

$$\theta=\cos^{-1}\frac{31}{5\sqrt{42}}$$

49. The corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5) then the minimum value of $z=4x+6y$ occurs at

1) finite number of points
2) infinite number of points
3) only one point
4) only two points

Ans. 4

Sol. At (0,2), (3, 0), z=12
Hence minimum at 2 points.

50. If A and B are two independent events such that $P(\bar{A})=0.75$, $P(A\cup B)=0.65$, and $P(B)=x$, then find the value of x

1) $\frac{5}{14}$ 2) $\frac{8}{15}$ 3) $\frac{9}{14}$ 4) $\frac{7}{15}$

Ans. 2

Sol. $P(A)=\frac{1}{4}$, $P(B)=\frac{3}{4}$, $P(A\cup B)=\frac{13}{20}$

$$\frac{1}{4}+x-\frac{1}{4}.x=\frac{13}{20}$$

$$\frac{3}{4}x-\frac{13}{20}-\frac{5}{20}=\frac{8}{20}$$

$$x=\frac{8}{20}\times\frac{4}{3}=\frac{8}{15}$$

51. Find the mean number of heads in three tosses of a fair coin

1) 1.5 2) 4.5 3) 2.5 4) 3.5

Ans. 1

Sol.

X	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{Mean }=\frac{3}{8}+\frac{6}{8}+\frac{3}{8}=\frac{12}{8}=\frac{3}{2}=1.5$$

52. If A and B are two events such that $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P\left(\frac{A}{B}\right)=\frac{1}{4}$, then $P(A'\cap B')$ is

1) $\frac{1}{4}$ 2) $\frac{3}{16}$ 3) $\frac{1}{12}$ 4) $\frac{3}{4}$

Ans. 1

Sol. $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$

$$P(A\cap B)=P(\overline{A\cup B})$$

$$P(A\cap B)=1-P(A\cup B)$$

$$P(A\cap B)=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]$$

$$P(A\cap B)=1-\left[\frac{6+4-1}{12}\right]$$

$$P(A\cap B)=1-\frac{9}{12}$$

$$P(A\cap B)=\frac{1}{4}$$

53. A pandemic has been spreading all over the world. The probabilities are 0.7 that there will be a lockdown, 0.8 that the pandemic is controlled in one month if there is a lockdown and 0.3 that it is controlled in one month if there is no lockdown. The probability that the pandemic will be controlled in one month is

1) 0.65 2) 1.65 3) 1.46 4) 0.46

Ans. 1

Sol. $P(E_1)=$ probability of there is lockdown =0.7

$P(E_2)=$ probability of there is lockdown=0.3

A is the event controlled in one month

$P(A/E_1)=0.8$, $P(A/E_2)=0.3$

$P(A)=0.7(0.8)+(0.3)(0.3)$

$=0.56+0.09=0.65$

54. The degree measure of $\frac{\pi}{32}$ is equal to

1) $5^0\ 30'\ 20''$ 2) $5^0\ 37'20''$

3) $5^0\ 37'30''$ 4) $4^0\ 30'\ 30''$

Ans. 3

Sol. $\frac{\pi}{32}=\frac{180^0}{32}=5^0 37'30''$

55. The value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ is

1) 0 2) 1 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

Ans. 4

Sol. $\sin\frac{5\pi}{12}.\sin\frac{\pi}{12}$

$=\frac{1}{2}\sin\frac{\pi}{6}$

$=\frac{1}{2}+\frac{1}{2}=\frac{1}{4}$

56. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=$

1) $\sin 2\theta$ 2) $2\cos\theta$

3) $2\sin\theta$ 4) $2\cos\frac{\theta}{2}$

Ans. 2

Sol. $1+\cos\theta=2\cos^2\left(\frac{\theta}{2}\right)$

$\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

57. If $A=\{1,2,3,.....10\}$ then the number of subsets of A containing only odd numbers is

1) 31 2) 27 3) 32 4) 30

Ans. 3

Sol. Odd number $=\{1,3,5,7,9\}$

No. of sub sets $=2^5=32$

58. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A\times B$ is 7 then $p^2+q^2=$ ______

1) 50 2) 51 3) 42 4) 49

Ans. 1

Sol. $n(A)=p,\ n(B)=q$

$n(A\times B)=7$

$pq=7$

$p^2+q^2=7^2+1^2$ or 1^2+7^2

$p^2+q^2=50$

59. The domain of the function $f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ is

1) $[-2,0)\cap(0,1)$ 2) $[-2,1)$

3) $[-2,1)$ 4) $[-2,0)\cup(0,1)$

Ans. 4

Sol. **1-x>0, 1-x** $\neq 1$

$x-1<0\quad x\neq 0\quad x+2\geq 0$

$x<1\quad x\geq -2$

$\therefore x\in[-2,0)\cup(0,1)$

60. The trigonometric function $y=\tan x$ in the II quadrant

1) Decreases form 0 to ∞

2) Decreases form $-\infty$ to 0

3) Increases from 0 to ∞

4) Increases from $-\infty$ to 0

Ans. 4

Sol. **By graph**